Modelling Emergent Crowd Behaviour

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Abstract. Using a cellular automata approach we create models of people movement within buildings. Simple, local update rules capturing the movement of individuals are shown to produce realistic behaviour of crowds, that is, collections of individuals. We demonstrate how distinct crowd behaviour at constriction points is characterised using different update rules. These distinct rules are produced in an experimental manner; we utilise a simulation environment to examine various crowd movement scenarios with the resulting crown dynamics being revealed by graphical animation.

Key words: Cellular Automata, Pedestrians, Simulation

1 Introduction

Pedestrian flow in a building requires careful design consideration to minimize potential bottlenecks that would increase both the evacuation time and the risk of crowd injury. Recent high profile disasters involving crowd evacuations bring into focus the importance of rigorous crowd flow analysis via modelling and simulation. These tragedies have mostly involved crowd crush or failure to escape a building due to congestion; however, pedestrian modelling need not be limited to just this scenario. Pedestrian movement is becoming an important factor in building design since engineers and architects alike need to ensure smooth crowd flow under varying crowd densities. This is important, since a building that provides smooth flow is likely to provide a safer and more comfortable environment.

We present techniques for modelling distinct crowd behaviour within a fixed structure using the cellular automata (CA) modelling paradigm, where space and time are both discretised [4, 18]. The CA approach permits us to develop simple automata update rules which, dependent on the state of neighbouring automata, capture the movement of individuals – the microscopic behaviour of the crowd system. From this simple microscopic behaviour we are able to demonstrate the realistic emergent macroscopic behaviour of a total system, that of the overall crowd, which results. In particular, we are able to demonstrate

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that slight differences to the CA update rules capture quite different crowd behaviour which is context dependent. For example, we are able to model crowds of people exiting a building in distinct situations such as orderly, unhurried exit versus an evacuation situation. A simulation environment which exercises our CA people movement model has been used experimentally to investigate how slight changes to the CA update rules may cause significant changes to the behaviour of the overall system. The patterns which emerge are revealed by the graphical animation component of the simulation environment. This is a feature classically found in complex systems; the emergent behaviour of the total system is frequently difficult to predict from knowledge of the local microscopic behaviour from which the system is constructed. Furthermore, small changes at the microscopic level may have substantial effects on the behaviour of the whole system.

Providing an environment that allows engineers to visually locate bottlenecks is clearly a valuable tool. The ability to view the evacuation from a bird's eye view can help in understanding the building dynamics. This visual overview allows one to "play games" with crowd densities and locations. An example is predicting how the timing of arrivals affects global flow, i.e. the boarding of two jumbo jets at an airport, or the arrival of two trains at a train station. Small increases in crowd density upstream may result in congested flow downstream, at critical areas.

We approach modelling people movement over a landscape using a technique for modelling traffic systems using the CA methodology [8–11, 14, 24]. Space is discretised into square cells which will be assumed to hold a single person. This approach is consistent with that adopted in the pedestrian modelling community [1, 2]. Each cell behaves as a simple finite state machine whose state changes dependent on both its current state and that of the adjacent cells which constitute its immediate neighbourhood. For a given direction of movement, determined by a Potential Field technique, person movement from one cell to another is determined by the ability of an occupied cell (one containing a person) to pass that person on to an unoccupied cell in a forward direction. Our CA modelling approach is related to that followed by Thompson, et al. [15–17], Galea, et al. [5] and Klüpfel and others [7]. In contrast to this related research we utilise very simple CA update rules, use a Potential Field technique to determine direction of movement and discover appropriate rules for capturing distinct movement behaviour determined by whether exiting is urgent or not.

Our use of the CA philosophy for modelling people movement is presented in Section 2. In Section 3 we introduce the Potential Field technique which is used to determine direction of movement. In Section 4 we describe a number of rule sets and discuss the particular macroscopic behaviour which emerges from them as a test scenario is simulated.

2 Pedestrians As Cellular Automata

Cellular Automata (CA) is a technique for modelling the dynamics of spatial systems, originally due to Ulam and von Neumann [18], where a discrete environment of cells is updated in steps, where for each step all the cells are updated according to some consistent global rules. A well know 'artificial' CA system is Conway's 'Game of Life' [6]; this system is composed of four simple rules describing how a cell survives according to the state of the cells around it. The resulting behaviour from such simple rules is surprisingly complex, and it is aptly termed a complex system. Pedestrian movement exhibits many of the properties of a complex system.

The physical environment is discretised as a grid of cells, one pedestrian may occupy one cell at a time [25]. Pedestrian movement is captured by the finite state machine behaviour associated with each cell; cells update dependent on their state and the state of their immediate neighbourhood.

The neighbourhood for each pedestrian is the surrounding grid extending out to a fixed radius; for simplicity our simulation only considers a radius of one cell.

It is the cells within this neighbourhood that are considered in the pedestrians update rules, and from this a move formulated. A simple example of a pedestrian ruleset could be as follows ('ahead' for a cell refers to the direction suggested by the Potential Field at that location).

- Check the cell directly ahead,
  - If this cell is empty then move into it
- Check diagonally ahead (either left or right) in turn
  - If this cell is empty then move into it
- Don't move

Ruleset 1: The simplest rule-set possible that results in a move being made is to *check* directly ahead and if this cell is empty, move into it – This is shown in Figure 1. Checking a cell, simply refers to an occupancy test on it. Ruleset 1 does not attempt to model individual destination information, since the move made is always in the suggested direction provided by the Potential Field.

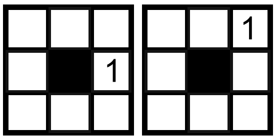

Fig. 1. The matrix graphically represents the order in which a pedestrians surrounding cells are considered. The pedestrian is located in the central black box, and is shown travelling to the right and diagonally right. A pedestrian will consider a cell to move into, and will only move if that cell is empty. Cells are considered in the order shown within the matrix.

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The behaviour emerging from this rule-set shows pedestrian queuing at the doorway as in Figure 3. Interestingly, as simple as this rule-set is, it relates closely to actual pedestrian phenomena. Similar behaviour to this may be seen in an orderly exit procedure such as when people leave a concert hall after a performance.

To demonstrate the effect of this update rule to a given population of pedestrians moving in a given direction, we utilise a specific scenario of a long hall with a constriction occurring halfway along it, pictured in Figures 2 and 3. This hall example was designed to test the behaviour of people moving through a narrow doorway. This scenario isolates a potential problem situation in most environments, that of the bottleneck at a door.

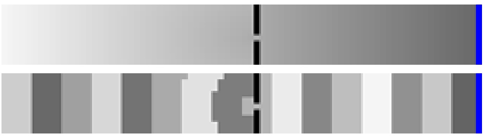

Fig. 2. The empty hall test environment and the corresponding Potential Field. The pedestrians flow from left to right

The test environment was set up as an 80 by 10 cell corridor (32m. by 4m.) with a partition dividing it into two. This partition has a single cell doorway that the pedestrians must move through. Each cell corresponds to a physical space of 0.4 meters square, traditionally taken to be that comfortably occupied by a single person [25]. The environment is seeded with 100 randomly distributed people, all of whom are moving to a line of attractors (a concept discussed in the next section) situated to the right of the environment.

Using this simple space and left-to-right crowd movement, we use our simulation environment to experiment with 3 rule-sets and observe the resulting emergent behaviour using the graphical animation feature of the simulation environment. A series of simulation experiments was run under identical starting conditions, with the rule-set changed each time; using Ruleset 1, Ruleset 2 and Ruleset 3 as introduced in Section 4. Each image (Figure 3) was taken at 15 step intervals giving a sequence of images picturing movement through time.

3 Determining Direction via Potential Fields

In order to model the flow of people through an environment, there must be some routing technique that determines the direction of the pedestrian flow. There are a number of ways to achieve such a routing effect; consider the routing differences based between agent based simulation and CA based simulation.

Typically in an agent based model, each entity has some agenda. This may be a list of way points, or destinations unique to that agent. In this scenario we

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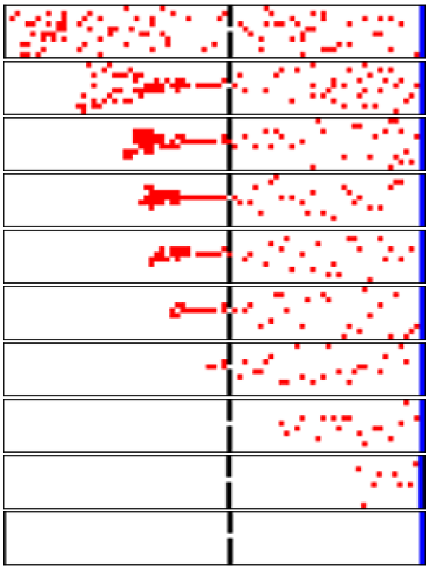

Fig. 3. Here the pedestrians are moving down the hall to the line of exits on the right, and each frame is taken at 15 step intervals. The movement is being described by Ruleset 1, and describes a 'queueing' behaviour similar to what one might expect to see of people leaving a concert hall.

can see that since each agent has a current list of targets, it can use information from the environment to locate and move toward each target in turn.

This approach is unsuitable for a CA model. The reason for this is the fundamental requirement of CA whereby each cell of the environment stores a simple value representing the state of its location. This simplicity is where CA gains its power, and to introduce any unnecessary complexity would detract from the rapid-simulation benefit arising from using the CA paradigm. We abstract as much information away from the pedestrian as possible and thus reduce the number of potential cell states.

Potential Fields provide such a means of extracting global routing knowledge. For every position in the environment there is a direction that provides the shortest route to the exit. A Potential Field is generated from the exit points and the shortest exit route found by choosing the direction from a pedestrian cell that has the lowest value on the Potential Field (See Figure 4.

Potential Field generation can be done in different ways, each with varying success. A simple approach involves each cell *flooding* its incremented count to all adjacent cells and $\sqrt{2}$ to all diagonal cells. The term flooding refers to recursively applying this operation to all surrounding cells. If a new value is flooded to another cell, the previous value is compared and the lesser of the two is stored. This recursive flooding gives an approximation to the shortest Euclidean distance

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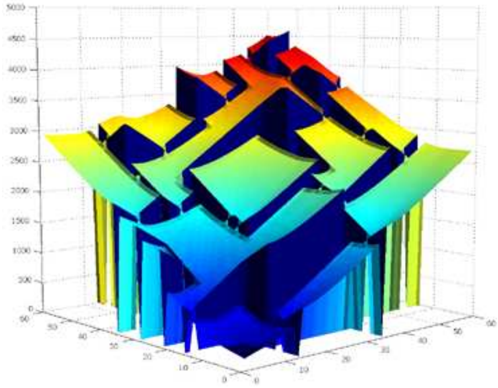

Fig. 4. A three dimensional representation of a complex Potential Field.

between the exit to any point in the environment (provided it is reachable) and has traditionally been used in robot path planning applications [3]. An example of this is when a robot is represented as a single point in a known environment and the shortest route to some destination is calculated using a Potential Field approach. This application is very similar to our needs for a pedestrian evacuation simulation.

Figure 5 (a) shows the rings of equidistance propagating from the exit on the right of the map.

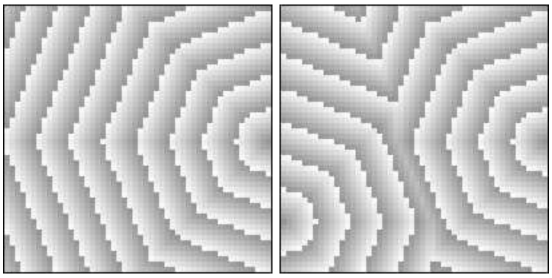
(a) (b)

Fig. 5. Here we see the rings of equal value (distance) propagating out from the source in (a), and how well Potential Fields apply to multiple exits (b). The lines represent contour information.

The Potential Field technique also provides a simple way to implement multiple exits. During the creation of the Potential Field, each exit can be flooded in turn and the resulting values at each point correspond to the shortest distance to any exit on the map. This routing technique stays true to how humans determine direction since it provides the direction to the closest exit if more than one exit is available. An example of this is shown in Figure 5 (b) where an extra exit has been added to the bottom left. A ridge is visible between each exit, this corresponds to the ridge of equi-distance. For any entity on this ridge, they will non-deterministically choose either direction thence we can determine the direction of movement for a person residing in a particular cell or location.

Perhaps the most powerful aspect of the Potential Field technique is in the way it deals with obstacle avoidance. Clearly if a cell contains an obstacle then the obstacle cell will not flood it's neighbours. As simple as this is, it restricts the propagation of the wave and allows refraction to occur around obstacles. This is important as it allows us to stop thinking in terms of 'walls' and 'obstacles' but to abstract a whole class of things as obstacles and reduce the number of states a cell in the environment can take (i. e. empty, obstacle, pedestrian). This means that walls are now essentially just a line of obstacles, an example of the effect of obstacles on a field is shown in Figure 6.

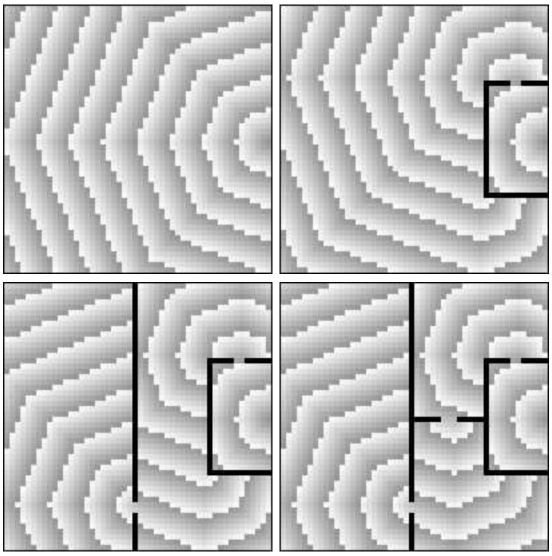

Fig. 6. The wave front diffraction propagates through the environment and simplifies routing for even the most complex environments.

4 Modelling Alternative Movement Characteristics

The orderly queuing pattern which emerged from Ruleset 1 and pictured in Figure 3 corresponds to the orderly pattern as seen at a venue ticket barrier or at the end of a concert. Disorderly queuing patterns which appear at a constriction

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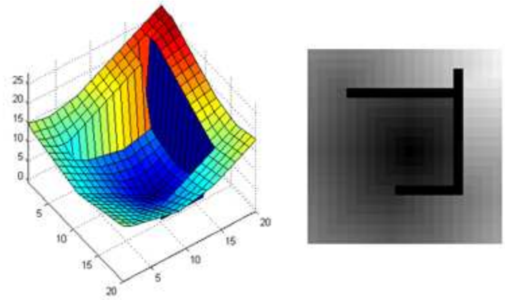

Fig. 7. The outward counting done during Potential Field generation can be represented in 3D shown on the left for a space with walls as shown and an exit at midpoint location (10,10).

can be found in other situations such as evacuation in an emergency or an exit from a soccer match where there is pushing and shoving. In different parts of the world different conventions are also adopted as to how orderly a queue is formed, if at all! Experimentation with alternative rules to Ruleset 1 has determined that these distinct patterns can be readily modelled.

Ruleset 2: For a large crowd to use Ruleset 1, one can see that there is not going to be any overtaking or jostling for position; this behaviour is not incorporated in Ruleset 1. This may now be modelled by checking diagonals after checking the forward direction. This allows pedestrians to overtake others and jostle for “better” position. That is, we model a pedestrian’s wish to move in a predefined direction. However, if the immediate adjacent cell is either occupied or is an obstacle, then another direction of movement is attempted. In Figures 1 and 8, the 1 indicates the first movement priority from the black occupied cell, moving to an attractor on the right. The 2 indicates the second priority choice for movement and the 3 in Figure 10 the third priority.

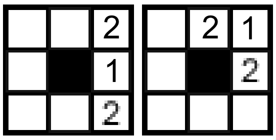

Fig. 8. Here forward checking is done first, followed by checks in either diagonal direction. The adjacent and diagonal masks are shown from left to right.

Considering diagonals after forward checking for occupancy resulted in crowd flow as shown in Figure 9. This diagonal movement breaks down the linear queuing that was evident in Ruleset 1. The result of the new Ruleset was the large oval shaped congregation of people at the door. This scenario visually resembles a less ordered evacuation of a building, or a less considerate crowd

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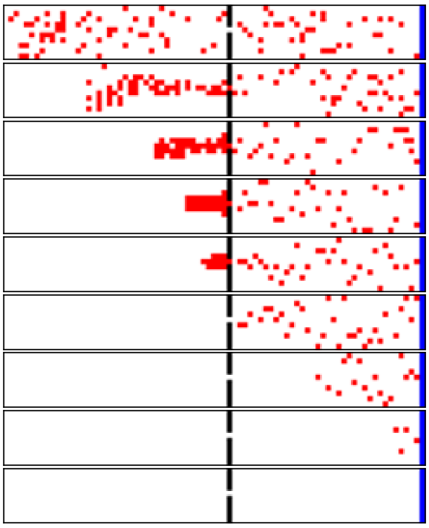

Fig. 9. The movement shown here is described by Ruleset 2. This rule provides ordered evacuation behaviour where queueing is still visible, however the line is far less ordered than described by Ruleset 1.

leaving a venue. There's an apparent lack of orderly queuing as it is evident that people are pushing past others to reach an exit.

Ruleset 3: The next rule development from Ruleset 2 is as follows: after all checks forward and diagonal fail, consider the cells on either side. This adds a more mobile element to the entity, due to the ability of a person to sidestep around crowds. This is likely to lend itself to a more agitated crowd, since sidestepping is likely to occur in very crowded situations.

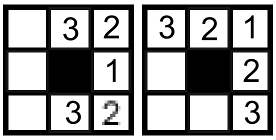

Fig. 10. Building upon Ruleset 2, we check side cells for occupancy after checking forward and diagonally.

We do not consider any movements where the person in a cell attempts to move backwards since this is an unrealistic scenario.

The results from Ruleset 3 (Figure 11), differ subtly to those from Ruleset 2. The shape of the congestion has changed from a pointed oval to a flattened circle

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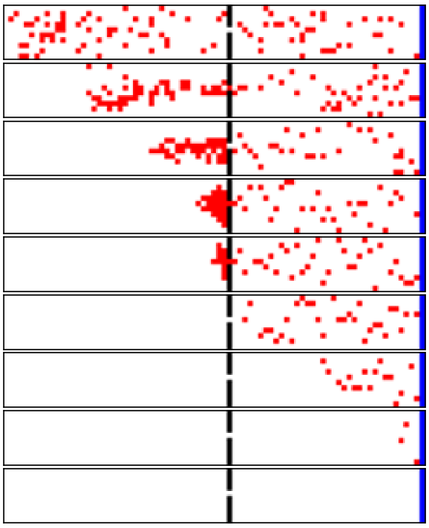

Fig. 11. The movement described by Ruleset 3 provides a more frantic evacuation model. The crowd is less ordered and there is a visible crush of people at the door trying to push themselves into the optimal position.

against the wall. This is indicative of a panic scenario where there is visible disorder in the crowd and the resulting flow is more mobile [1,2].

5 Concluding Remarks

We have illustrated how simple CA rules are used to capture people movement and model the emergent behaviour of a crowd as it moves through a given space. A Potential Field technique is used to determine the direction in which the crowd is "pulled" and a time-lapse graphical display is used to picture the simulated crowd dynamics, as the simulator runs the CA model. We have demonstrated how simple differences in the update rules capture distinct movement patterns which we believe characterise distinct crowd behaviour dependent on situation (e.g. evacuation versus orderly exit) or differences in queuing conventions. These distinct, emergent crowd behaviour patterns have been determined experimentally using a CA simulation environment and their realism determined from the graphical display by observation.

Our simulation environment, and the CA models which it exercises, may be readily used to determine crowd dynamics within more complex structures than the long hall example used here to illustrate the distinct macroscopic behaviour which results from different Rulesets. Figure 12 pictures the emergency crowd flow patterns which occur when a series of interconnected spaces all utilise a

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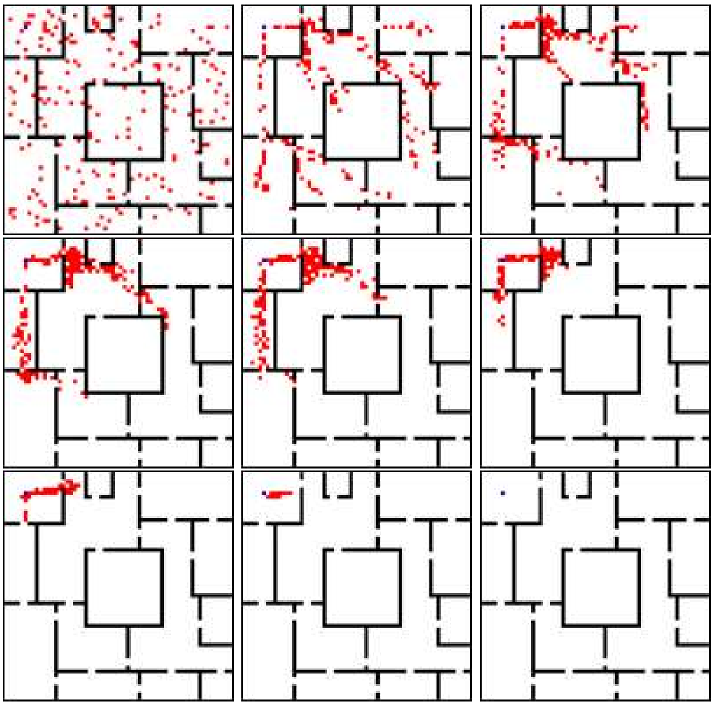

Fig. 12. Pedestrian flow according to ruleset 3 is shown with 10 time steps between each frame. Frames are ordered from top left down to bottom right, and the exit is located in the top left corner of the map.

single exit in the middle of the top leftmost room. Figure 13 illustrates the Potential Field which result in the appropriate direction of flow required to exit each room space.

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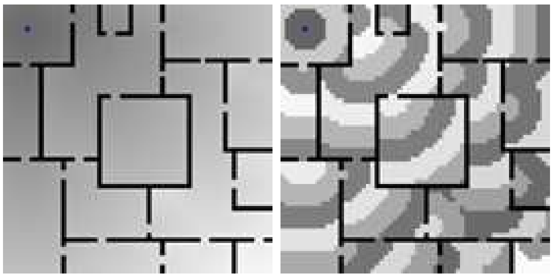

Fig. 13. The empty office floor and the corresponding Potential Field. The Potential Field has been thresholded to aid its visualisation

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